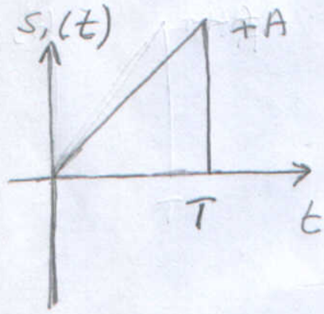
Çankaya University – ECE Department – ECE 376

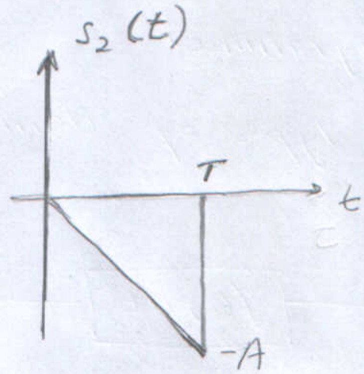
Student Name :
Student Number :

Duration : 2 hours
Open book exam

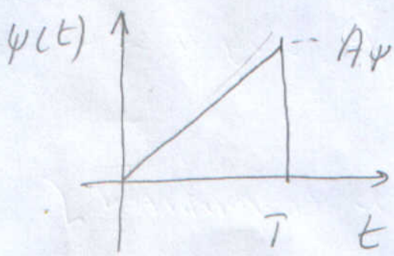
Questions

1. (50 Points) The two signals $s_1(t)$, $s_2(t)$ are given as below. Determine what type of modulation (i.e., ASK, PSK, QAM, FSK), these signals represent and their dimensionality. Find the set of $\psi_k(t)$ orthogonal basis functions for $s_1(t)$, $s_2(t)$. Draw the signal constellation diagramme, showing the position, the length of vectors **s₁**, **s₂** and the distance between them. For these signals, draw separately the appropriate receiver diagrammes consisting of correlator type demodulator and matched filter. For the case of matched filter, find the output if $s_1(t)$ was sent from the transmitter.





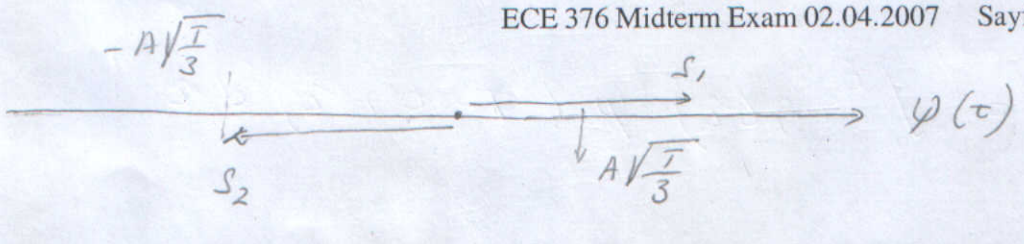
Solution: Since one orthogonal basis function is sufficient to represent both $s_1(t)$ and $s_2(t)$, the given signals constitute ASK, thus one dimensional, that is $M=2$, $N=1$.



From the condition $\int_0^T \varphi^2(t)\,dt = 1$

$$A_\varphi = \sqrt{\frac{3}{T}}, \quad \varphi(t) = \frac{t}{T}\sqrt{\frac{3}{T}}$$

$$\text{Energy of } \frac{s_1(t)}{s_2(t)} = \int_0^T s_1^2(t)\,dt = \int_0^T s_2^2(t)\,dt = \frac{A^2 T}{3}$$



$s_1(t)$ and $s_2(t)$ in terms of $\psi(t)$

$$s_1(t) = A\sqrt{\frac{T}{3}}\,\psi(t) \quad , \quad s_2(t) = -A\sqrt{\frac{T}{3}}\,\psi(t)$$

vector

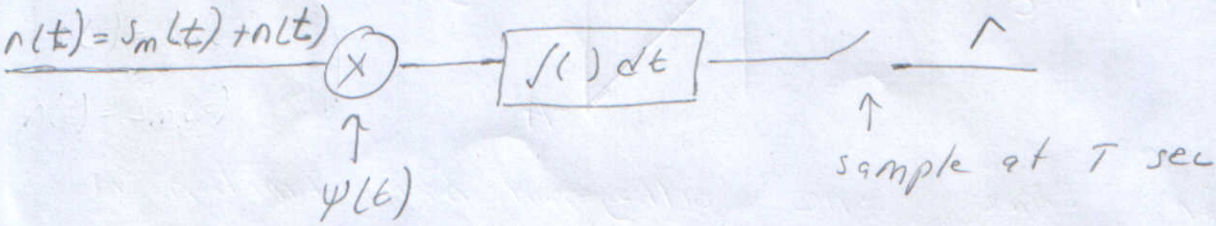
$$s_1 = \left[A\sqrt{T/3} \right] \quad , \quad s_2 = \left[-A\sqrt{T/3} \right]$$

Distance between s_1 and s_2 vectors is

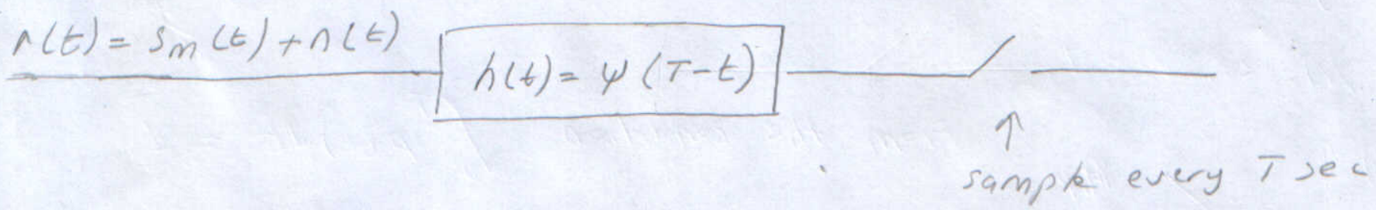
$$\| s_1 - s_2 \| = 2A\sqrt{T/3}$$

Receiver Diagramme

a) Correlator type of demodulator



b) Matched filter type of demodulator



Output of MF detector [if $s_1(t)$ is transmitted]

$$y(t) = \int_0^t s_1(\tau)\, h(t-\tau)\, d\tau + \int_0^t n(\tau)\, h(t-\tau)\, d\tau$$

signal part

$$y_s(t) = \int_0^t s_1(\tau)\, \psi(T-t+\tau)\, d\tau$$

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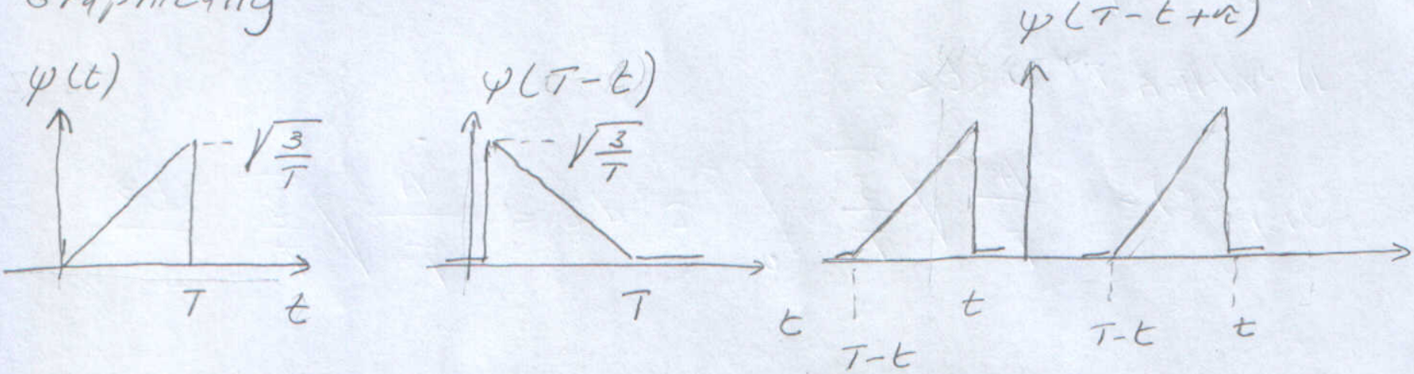
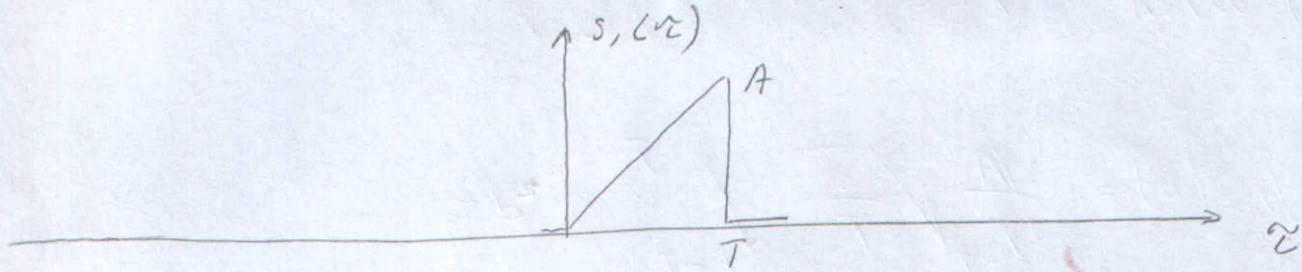
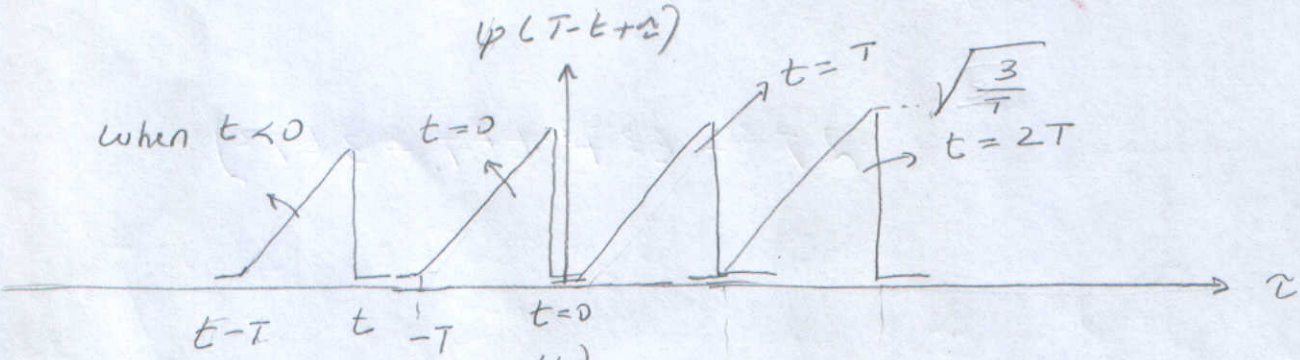
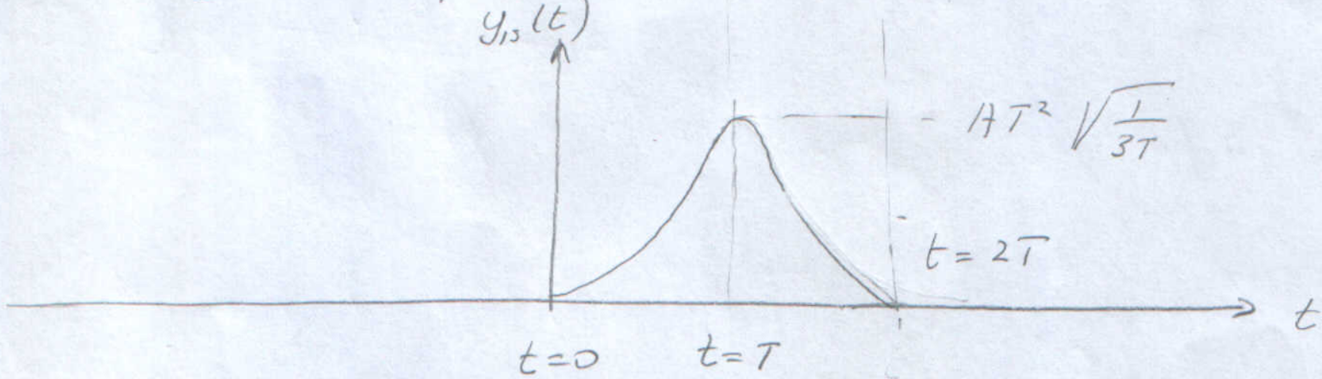
Graphically









As seen from above the lower end of $\psi(T-t+\tau)$ is marked as $t-T$, upper end is t. Accordingly if $s_1(t)$ was transmitted, overlap will begin at

$t = 0$ and terminate at $t = 2T$

Thus for the two regions

1) When $0 < t < T$

$$y_{1s1}(t) = \frac{A}{T}\sqrt{\frac{3}{T}} \int_0^t \tau^2 d\tau = \frac{A}{T}\sqrt{\frac{3}{T}} \frac{t^3}{3}$$

2) when $T < t < 2T$

$$y_{1s2}(t) = \frac{A}{T}\sqrt{\frac{3}{T}} \int_{t-T}^{T} \tau^2 d\tau$$

$$= \frac{A}{3T}\sqrt{\frac{3}{T}} \left(2T^3 - t^3 + 3t^2 \cdot T - 3tT^2 \right)$$

we see that when $t = T$

$$y_{1s1}(t = T) = y_{1s2}(t = T) = AT^2\sqrt{\frac{1}{3T}}$$

and at $t = 2T$

$$y_{1s2}(t = 2T) = 0$$

2. (20 Points) A message signal, $m(t)$ occupying a bandwidth of 3 kHz is modulated in DSB format and transmitted. The received power of this signal is 1μW. Assuming AWGN with noise spectral density $N_0 = 10^{-10}$ W/Hz, write the expression for the received signal plus noise and calculate the signal to noise ratio (SNR) at the input to receiver.

Solution:

$$r(t) = A_c\, m(t) \cos 2\pi f_c t + n(t)$$

Where $A_c^2 P_m = 2 \times P_R = 2 \times 1\mu W$ and $N_0 = 10^{-10} W/Hz$

Hence noise power at input to receiver

$$P_n = 2 \times \int_{-3kHz}^{+3kHz} \frac{N_0}{2}\, df = 10^{-10} \times 6 \times 10^3 = 6 \times 10^{-7} \text{ Watt}$$

$$SNR = \frac{\overbrace{2 \times 10^{-6}}^{A_c^2 P_m}}{\underbrace{6 \times 10^{-7}}_{2WN_0}} = \frac{10}{3} \simeq 3.34 \quad \text{or} \quad 5.23\, dB$$

The above is from Eq (5.1.13) of Proakis 2002

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason

a) In FM, the time derivative of the message signal varies the phase of the carrier : False

The (time) integral of message signal varies the phase of the carrier.

b) In FSK, M (number of possible signals to be transmitted) = N (Dimensionality of signal space) : True

c) Matched filter has a time response of $s_m(T-t)$ when $s_m(t)$ is one of the signals to be transmitted : False

Time response of MF is $h_k(t) = \psi_k(T-t)$ (k th branch)

(Partially)

d) In QAM, signal amplitudes vary with time during one symbol period : True but this is not the essential requirement to establish QAM

e) PSK can have one to three dimensions : False

PSK is only two dimensional